Highest Posterior Model Computation and Variable Selection via the Simulated Annealing – Supplementary Material

1. MATHEMATICAL RESULT

Below are the conditions which are the basis of the convergence of the SA-HPM method.

Condition 1. *The probability of moving to j th model from i th model in p steps is positive, that is, $q_{ij}^{(p)} > 0$.*

Condition 2. $q_{ii} > 0$.

Condition 3. *Q is irreducible.*

[**?**] showed that if the above conditions are met, then the simulated annealing method converges. Hence, the SA-HPM algorithm in Figure 1 in the paper, converges. The following lemmas together with the following proposition establish that the conditions are satisfied for our proposed SA-HPM algorithm.

Proposition 1. *The model space $\mathcal{M}$ is finite.*

Lemma 1. *In the SA-HPM algorithm, the probability of moving to j th model from i th model in p steps is positive, that is, $q_{ij}^{(p)} > 0$.*

Lemma 2. *In the SA-HPM algorithm $q_{ii} > 0$.*

Proof. Suppose at time t the i th model gets selected. Since the i th model itself belongs to the neighborhood of i th model according to our neighborhood region selection, we have,

$$q_{ii} = \frac{\text{posterior probability of } i\text{th model}}{\text{sum of posterior probabilities of neighbors of } i \text{ th model}} = \frac{P(\mathcal{M}_i)P(y|\mathcal{M}_i)}{\sum_{\gamma \in nbd(i)} P(\mathcal{M}_\gamma)P(y|\mathcal{M}_\gamma)},$$

By definition prior probability of any model is strictly greater than 0 and posterior probabilities are never 0 in practice for any given proper prior. This completes the proof. □

Lemma 3. *In the SA-HPM algorithm, the transition matrix Q is irreducible.*

Proof. We note that, each model has its own binary representation using γ. Given two binary sequences, they are neighbors within a finite number of transformations of the coordinates which is given by either $0 \to 1$ or $1 \to 0$. Hence, $\Pr(\mathcal{M}_j|\mathcal{M}_i) > 0$ for all $i, j \in \mathcal{M}$, trivially completing the proof. □

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