Model	Functional form $f(d, \theta)$	Model specification
linear	$E_0 + \delta d$	$E_0 = 0,\ \delta = 0.5$
emax1	$E_0 + E_{\max} d/(ED_{50} + d)$	$E_0 = 0,\ E_{\max} = 0.75,\ ED_{50} = 0.5$
emax2		$E_0 = 0,\ E_{\max} = 0.5321,\ ED_{50} = 0.0642$
exponential1	$E_0 + E_1(\exp(d/\delta) - 1)$	$E_0 = 0,\ E_1 = 0.00125,\ \delta = 0.1669$
exponential2		$E_0 = 0,\ E_1 = 0.3492,\ \delta = 0.3664$
quadratic1	$E_0 + \beta_1 d + \beta_2 d^2$	$E_0 = 0,\ \beta_1 = 1.70,\ \beta_2 = -1.445$
quadratic2		$E_0 = 0,\ \beta_1 = 1.4286,\ \beta_2 = -1.0204$
logistic1	$E_0 + E_{\max}/(1 + \exp((ED_{50} - d)/\delta))$	$E_0 = -0.0001,\ E_{\max} = 0.5116,\ ED_{50} = 0.6839,\ \delta = 0.0837$
logistic2		$E_0 = -0.0264,\ E_{\max} = 0.5701,\ ED_{50} = 0.5485,\ \delta = 0.1813$
power	$E_0 + E_{\max} d^h$	$E_0 = 0,\ E_{\max} = 0.5,\ h = 0.5$
sigmoid Emax(sigEmax)	$E_0 + E_{\max} d^h/(ED_{50}^h + d^h)$	$E_0 = 0,\ E_{\max} = 0.5146,\ ED_{50} = 0.2561,\ h = 2.5922$
beta model (betaMod)	$E_0 + E_{\max} B(\delta_1, \delta_2)(d/scal)^{\delta_1}(1 - d/scal)^{\delta_2}$	$E_0 = 0,\ E_{\max} = 0.5,\ \delta_1 = 0.3518,\ \delta_2 = 1.0554,\ scal = 1.2$

Table S1: Model specifications for the dose-response shapes selected for the true underlying dose-response model. For the beta model, $B(\delta_1, \delta_2) = (\delta_1+\delta_2)^{\delta_1+\delta_2}/(\delta_1^{\delta_1}\delta_2^{\delta_2})$.

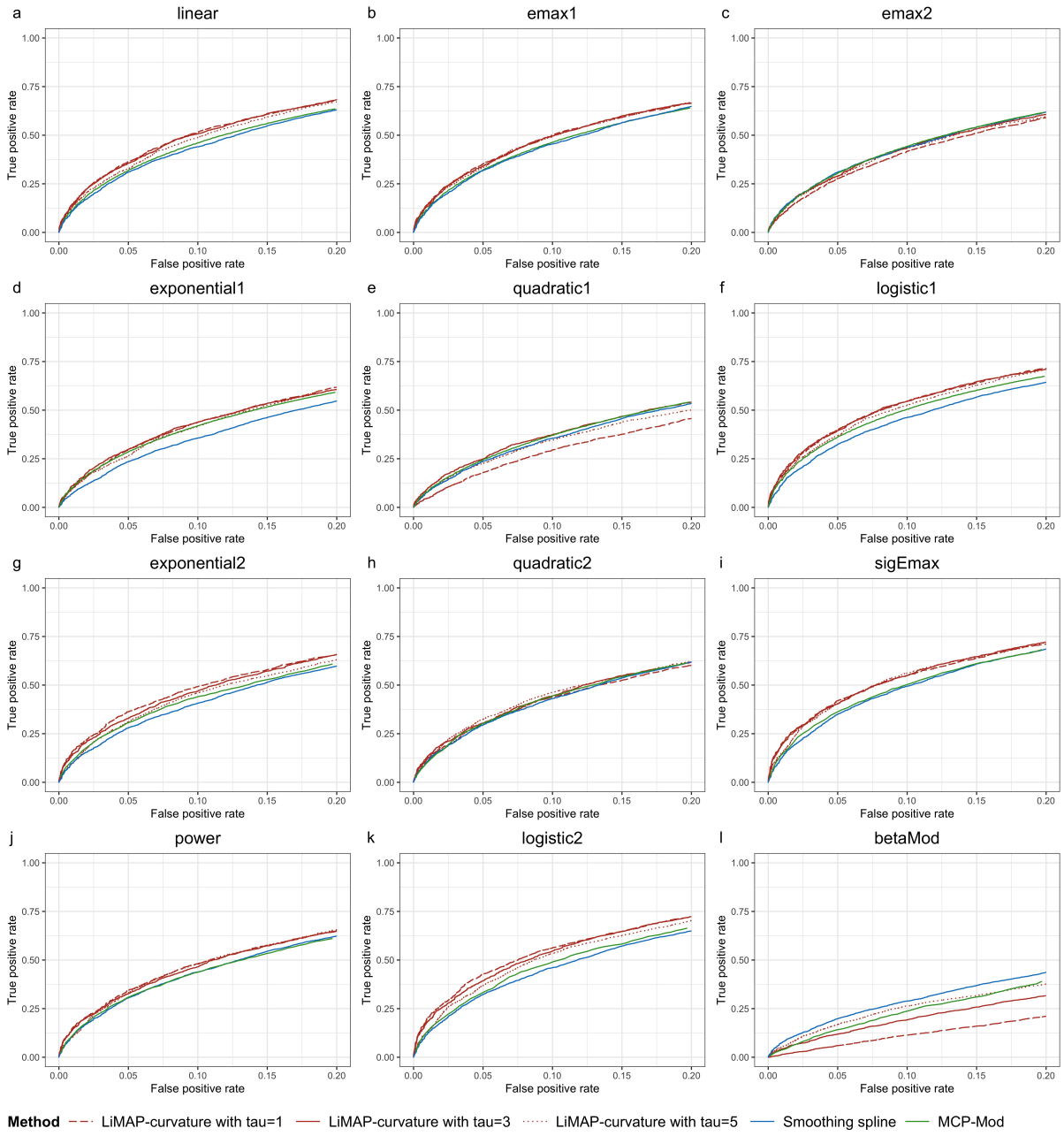

Figure S1: ROC curves of LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 10 patients per arm. The ROC curves of MCP-Mod in (a)-(f) are produced with the true underlying dose-response model not included in the candidate model set, and the ROC curves of MCP-Mod in (g)-(l) are produced with the true underlying dose-response model not included in the candidate model set.

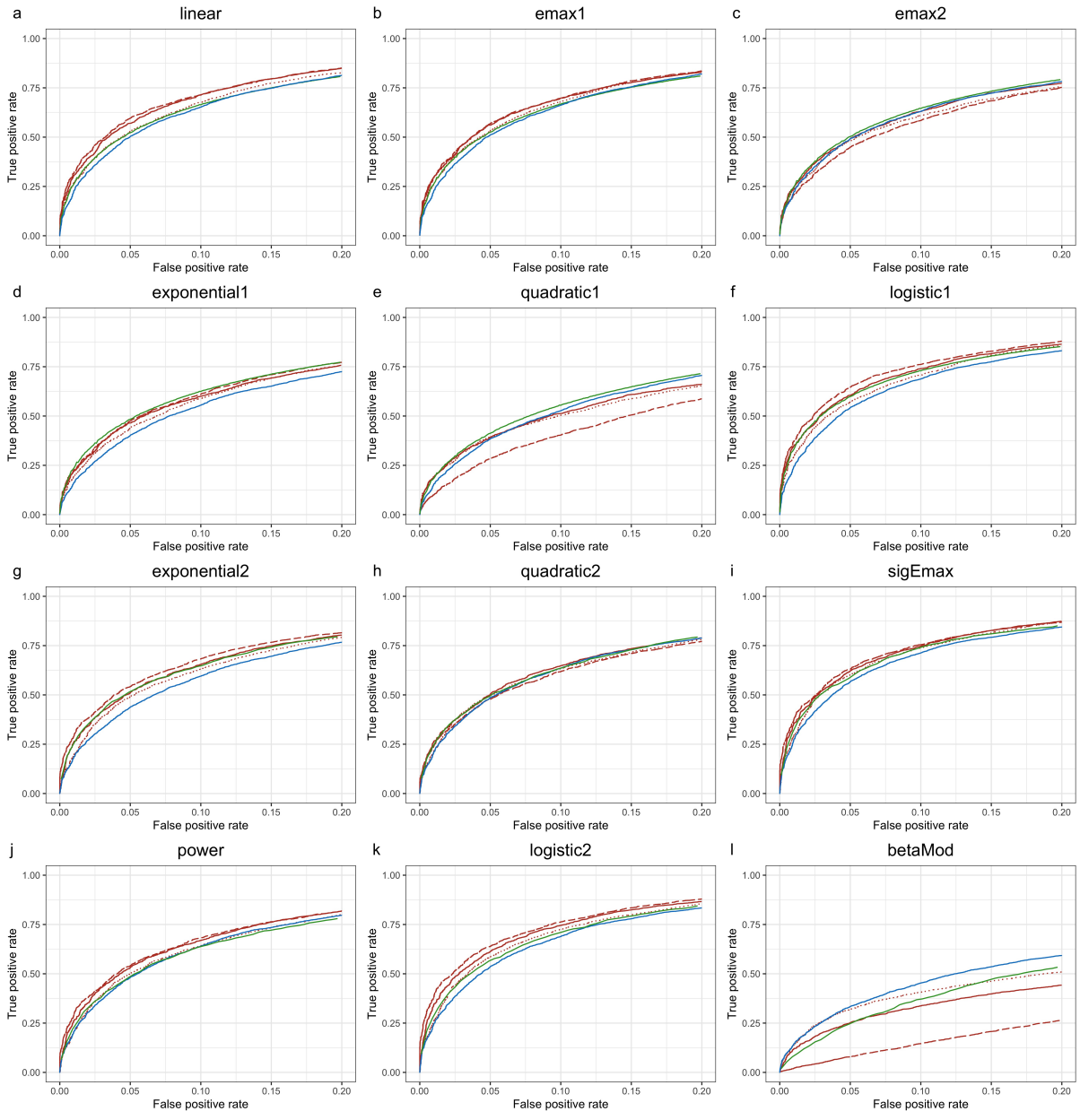

Figure S2: ROC curves of LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 20 patients per arm. The ROC curves of MCP-Mod in (a)-(f) are produced with the true underlying dose-response model not included in the candidate model set, and the ROC curves of MCP-Mod in (g)-(l) are produced with the true underlying dose-response model not included in the candidate model set.

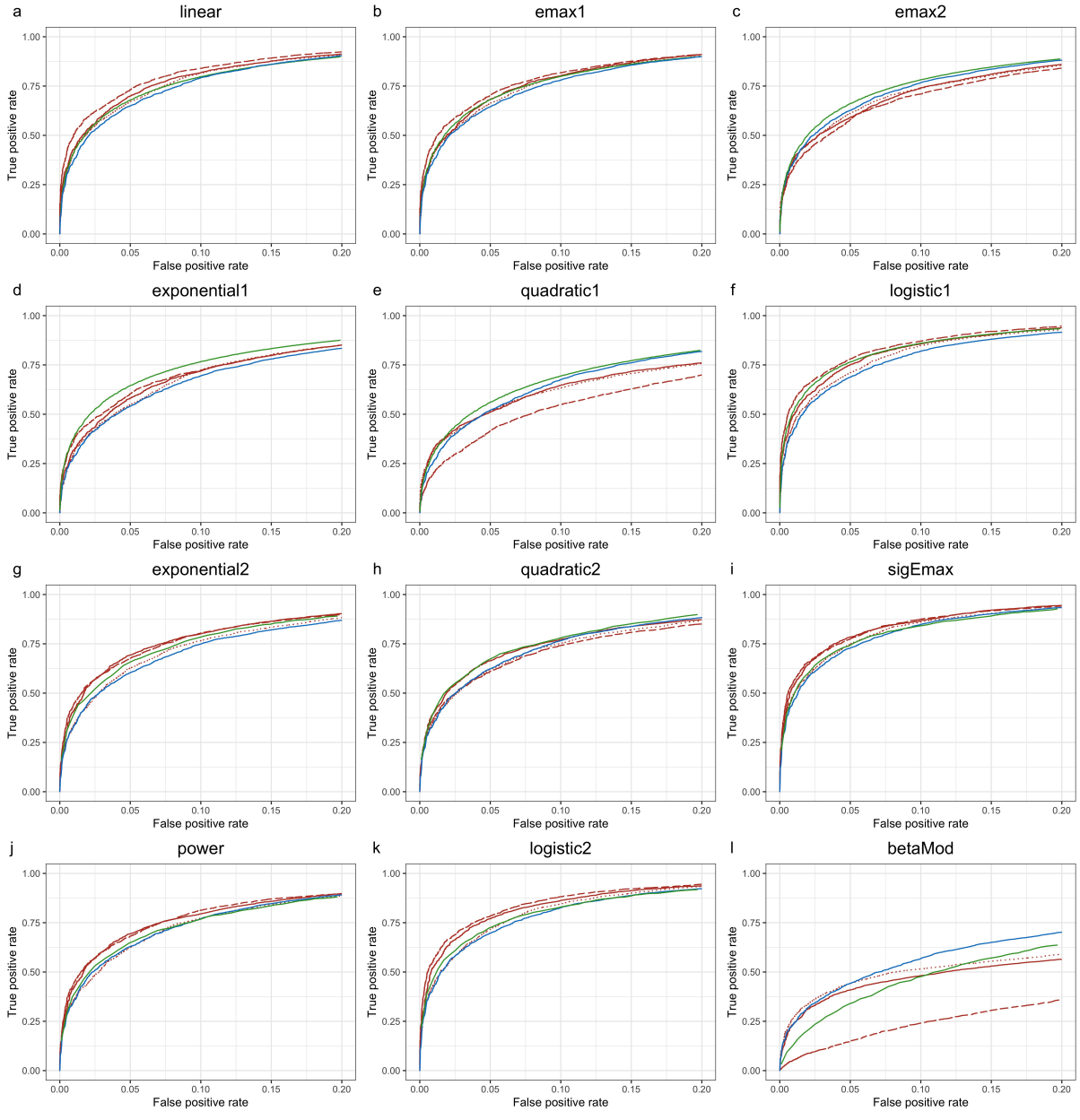

Figure S3: ROC curves of LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 30 patients per arm. The ROC curves of MCP-Mod in (a)-(f) are produced with the true underlying dose-response model not included in the candidate model set, and the ROC curves of MCP-Mod in (g)-(l) are produced with the true underlying dose-response model not included in the candidate model set.

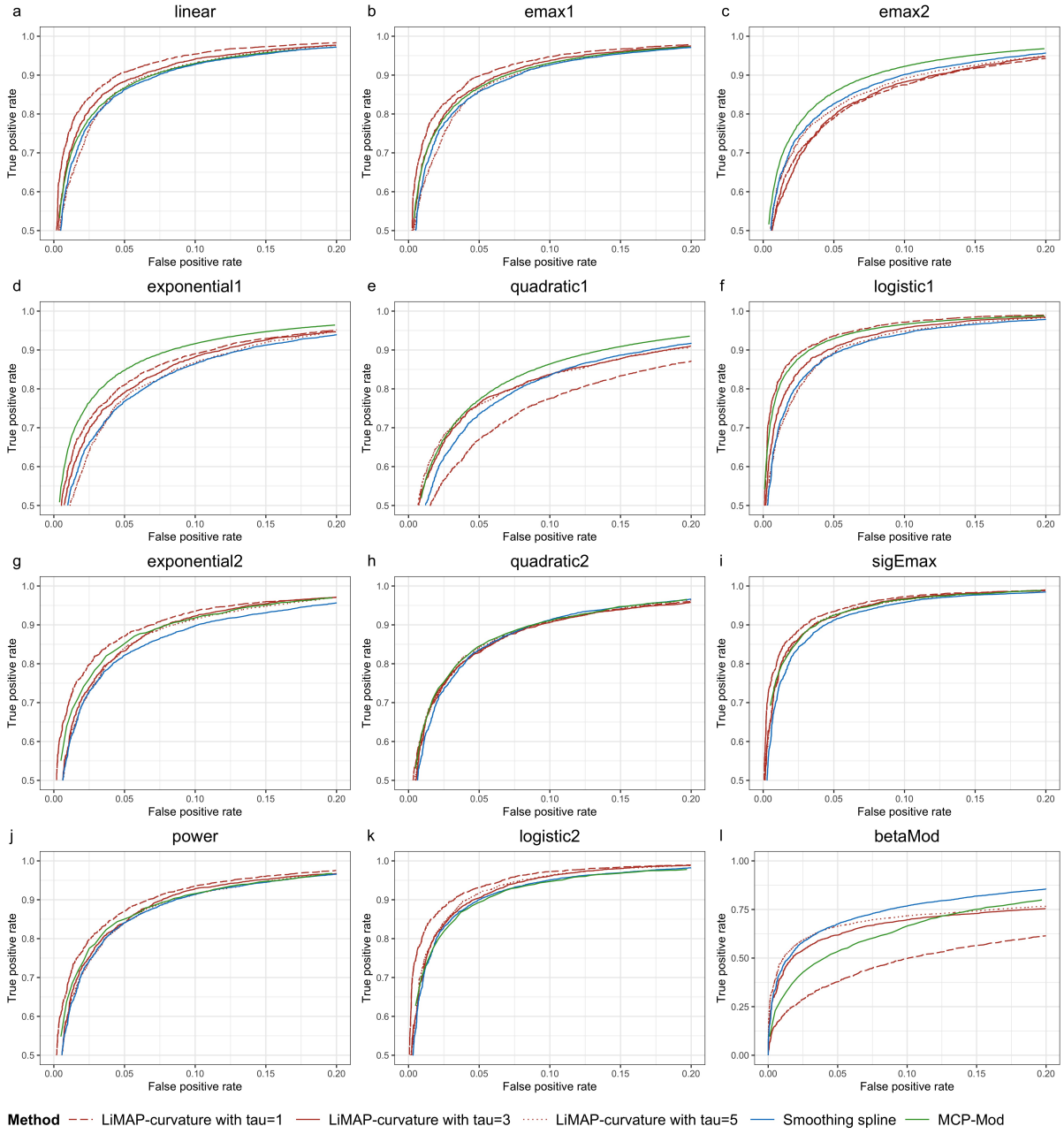

Figure S4: ROC curves of LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 50 patients per arm. The ROC curves of MCP-Mod in (a)-(f) are produced with the true underlying dose-response model not included in the candidate model set, and the ROC curves of MCP-Mod in (g)-(l) are produced with the true underlying dose-response model not included in the candidate model set.

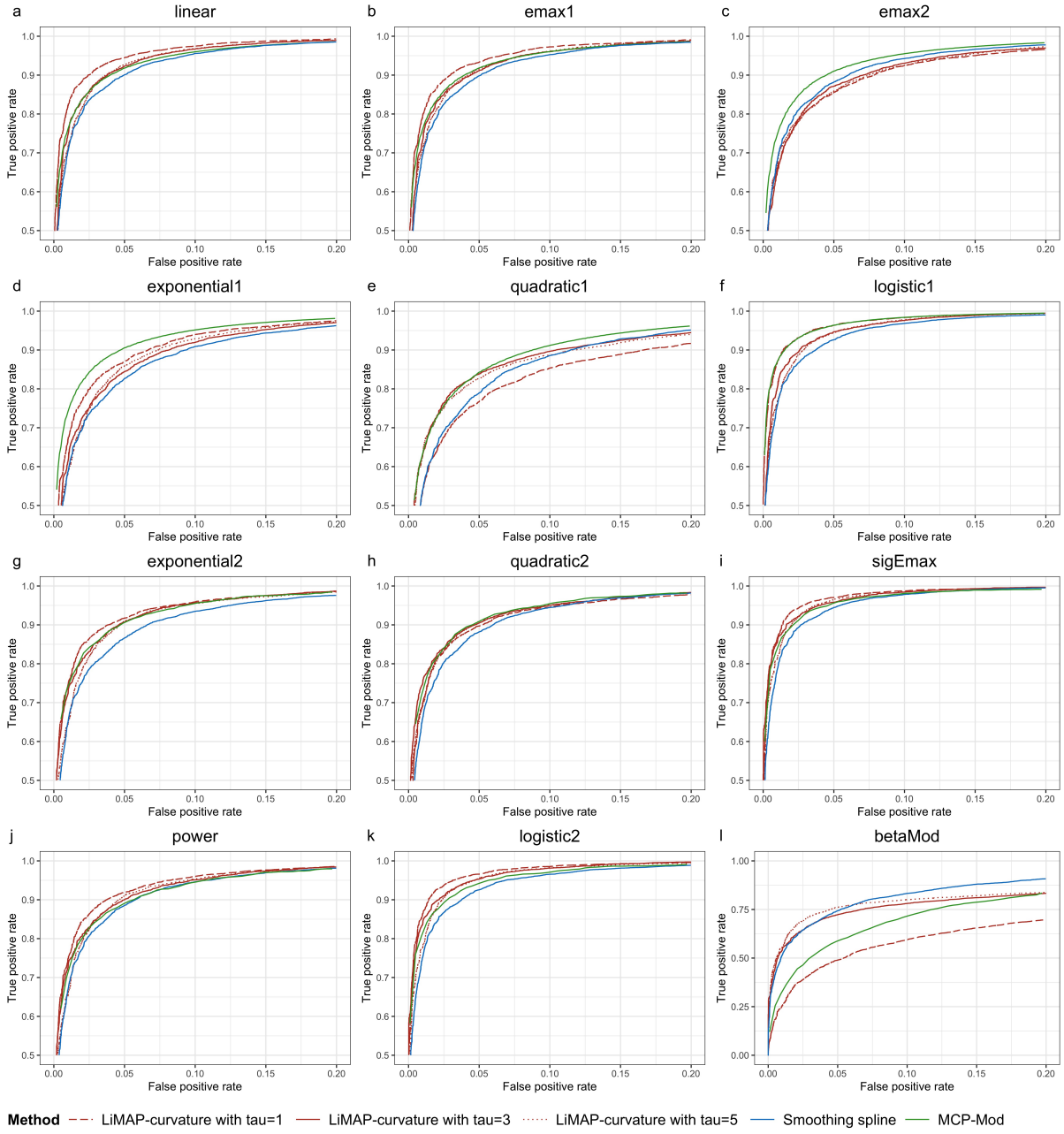

Figure S5: ROC curves of LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 60 patients per arm. The ROC curves of MCP-Mod in (a)-(f) are produced with the true underlying dose-response model not included in the candidate model set, and the ROC curves of MCP-Mod in (g)-(l) are produced with the true underlying dose-response model not included in the candidate model set.

Sample size	Model	LiMAP-curvature			Smoothing spline	MCP-Mod
		$\tau = 1$	$\tau = 3$	$\tau = 5$		
$N = 10$	linear	0.376	0.372	0.338	0.310	0.317
	emax1	0.360	0.370	0.327	0.317	0.319
	emax2	0.280	0.302	0.290	0.309	0.302
	exponential1	0.296	0.304	0.266	0.234	0.284
	quadratic1	0.182	0.233	0.244	0.236	0.243
	logistic1	0.418	0.411	0.357	0.322	0.359
	exponential2	0.361	0.336	0.300	0.279	0.299
	quadratic2	0.306	0.324	0.318	0.294	0.282
	sigEmax	0.411	0.414	0.388	0.349	0.347
	power	0.355	0.331	0.313	0.197	0.307
	logistic2	0.429	0.396	0.374	0.322	0.342
	betaMod	0.066	0.121	0.166	0.197	0.146
$N = 20$	linear	0.611	0.586	0.544	0.503	0.523
	emax1	0.582	0.553	0.548	0.511	0.525
	emax2	0.464	0.473	0.491	0.492	0.503
	exponential1	0.497	0.465	0.443	0.401	0.484
	quadratic1	0.446	0.519	0.524	0.384	0.413
	logistic1	0.656	0.629	0.563	0.542	0.598
	exponential2	0.546	0.541	0.494	0.438	0.488
	quadratic2	0.498	0.531	0.500	0.485	0.521
	sigEmax	0.628	0.634	0.624	0.566	0.575
	power	0.558	0.549	0.498	0.481	0.501
	logistic2	0.658	0.624	0.585	0.536	0.560
	betaMod	0.082	0.252	0.306	0.334	0.250
$N = 30$	linear	0.742	0.728	0.659	0.649	0.681
	emax1	0.719	0.699	0.656	0.645	0.683
	emax2	0.589	0.591	0.634	0.626	0.662
	exponential1	0.622	0.604	0.585	0.540	0.646

Table S2 continued from previous page

	quadratic1	0.537	0.651	0.663	0.520	0.562
	logistic1	0.781	0.774	0.740	0.688	0.766
	exponential2	0.697	0.664	0.622	0.603	0.663
	quadratic2	0.621	0.650	0.638	0.623	0.660
	sigEmax	0.779	0.766	0.748	0.728	0.731
	power	0.685	0.661	0.640	0.629	0.657
	logistic2	0.780	0.754	0.717	0.696	0.711
	betaMod	0.156	0.404	0.439	0.442	0.332
$N = 40$	linear	0.839	0.819	0.786	0.755	0.788
	emax1	0.884	0.799	0.777	0.768	0.791
	emax2	0.685	0.710	0.722	0.738	0.774
	exponential1	0.724	0.709	0.698	0.655	0.763
	quadratic1	0.547	0.652	0.676	0.622	0.678
	logistic1	0.886	0.851	0.827	0.795	0.867
	exponential2	0.804	0.774	0.747	0.711	0.777
	quadratic2	0.738	0.778	0.761	0.727	0.752
	sigEmax	0.889	0.878	0.858	0.832	0.830
	power	0.809	0.786	0.760	0.740	0.779
	logistic2	0.891	0.863	0.843	0.812	0.826
	betaMod	0.271	0.532	0.562	0.555	0.424
$N = 50$	linear	0.913	0.868	0.864	0.862	0.867
	emax1	0.888	0.867	0.863	0.857	0.868
	emax2	0.773	0.790	0.797	0.825	0.856
	exponential1	0.803	0.786	0.773	0.767	0.848
	quadratic1	0.651	0.757	0.763	0.734	0.773
	logistic1	0.931	0.911	0.898	0.890	0.929
	exponential2	0.873	0.832	0.823	0.821	0.870
	quadratic2	0.842	0.830	0.836	0.836	0.853
	sigEmax	0.942	0.920	0.916	0.912	0.911

Table S2 continued from previous page

	power	0.880	0.834	0.830	0.841	0.838
	logistic2	0.939	0.906	0.902	0.900	0.900
	betaMod	0.384	0.626	0.665	0.674	0.525
$N = 60$	linear	0.938	0.936	0.913	0.906	0.916
	emax1	0.933	0.923	0.916	0.902	0.917
	emax2	0.923	0.937	0.927	0.890	0.909
	exponential1	0.873	0.857	0.855	0.830	0.904
	quadratic1	0.759	0.839	0.841	0.798	0.841
	logistic1	0.966	0.948	0.946	0.930	0.963
	exponential2	0.917	0.891	0.888	0.872	0.903
	quadratic2	0.900	0.902	0.885	0.877	0.903
	sigEmax	0.966	0.958	0.950	0.951	0.946
	power	0.919	0.896	0.887	0.740	0.898
	logistic2	0.960	0.947	0.948	0.932	0.943
	betaMod	0.498	0.721	0.755	0.740	0.586

Table S2: Power values of LiMAP-curvature, smoothing spline and MCP-Mod to test the PoC across different true underlying dose-response models and sample sizes by controlling the type I error rate at 5%.

Sample size	Model	LiMAP-curvature			Smoothing spline	MCP-Mod
		$\tau = 1$	$\tau = 3$	$\tau = 5$		
$N = 10$	linear	0.516	0.501	0.497	0.440	0.460
	emax1	0.505	0.490	0.494	0.455	0.463
	emax2	0.415	0.411	0.442	0.439	0.442
	exponential1	0.438	0.407	0.415	0.356	0.419
	quadratic1	0.295	0.332	0.379	0.355	0.370
	logistic1	0.558	0.532	0.525	0.462	0.504
	exponential2	0.493	0.453	0.463	0.407	0.435
	quadratic2	0.438	0.437	0.451	0.430	0.432
	sigEmax	0.543	0.534	0.550	0.494	0.507
	power	0.474	0.469	0.475	0.438	0.428
	logistic2	0.544	0.520	0.529	0.460	0.489
	betaMod	0.113	0.191	0.252	0.289	0.257
$N = 20$	linear	0.775	0.706	0.699	0.652	0.663
	emax1	0.718	0.695	0.693	0.661	0.667
	emax2	0.605	0.619	0.623	0.630	0.645
	exponential1	0.638	0.611	0.588	0.554	0.624
	quadratic1	0.446	0.519	0.524	0.526	0.554
	logistic1	0.778	0.755	0.712	0.687	0.729
	exponential2	0.680	0.675	0.653	0.595	0.638
	quadratic2	0.628	0.649	0.640	0.635	0.657
	sigEmax	0.753	0.756	0.759	0.712	0.715
	power	0.685	0.679	0.655	0.640	0.646
	logistic2	0.775	0.747	0.727	0.688	0.717
	betaMod	0.158	0.331	0.404	0.453	0.359
$N = 30$	linear	0.845	0.822	0.806	0.792	0.797
	emax1	0.825	0.820	0.799	0.780	0.799
	emax2	0.716	0.738	0.734	0.767	0.781
	exponential1	0.732	0.729	0.713	0.694	0.766

Table S3 continued from previous page

	quadratic1	0.550	0.632	0.652	0.676	0.693
	logistic1	0.884	0.861	0.842	0.820	0.859
	exponential2	0.809	0.781	0.770	0.749	0.779
	quadratic2	0.758	0.759	0.768	0.765	0.784
	sigEmax	0.873	0.858	0.866	0.848	0.839
	power	0.780	0.778	0.786	0.770	0.776
	logistic2	0.875	0.860	0.855	0.826	0.827
	betaMod	0.233	0.461	0.520	0.568	0.471
$N = 40$	linear	0.911	0.904	0.885	0.866	0.880
	emax1	0.892	0.890	0.880	0.875	0.881
	emax2	0.819	0.821	0.808	0.846	0.868
	exponential1	0.826	0.829	0.823	0.789	0.858
	quadratic1	0.668	0.769	0.752	0.753	0.793
	logistic1	0.938	0.933	0.912	0.890	0.929
	exponential2	0.888	0.875	0.859	0.890	0.869
	quadratic2	0.839	0.863	0.853	0.845	0.856
	sigEmax	0.940	0.937	0.928	0.916	0.920
	power	0.891	0.876	0.869	0.898	0.851
	logistic2	0.938	0.933	0.916	0.853	0.908
	betaMod	0.367	0.608	0.626	0.679	0.560
$N = 50$	linear	0.957	0.939	0.932	0.927	0.929
	emax1	0.947	0.934	0.931	0.926	0.930
	emax2	0.872	0.883	0.879	0.901	0.921
	exponential1	0.893	0.885	0.879	0.864	0.915
	quadratic1	0.767	0.831	0.836	0.834	0.863
	logistic1	0.969	0.960	0.952	0.942	0.965
	exponential2	0.931	0.922	0.908	0.898	0.936
	quadratic2	0.907	0.910	0.908	0.912	0.923
	sigEmax	0.974	0.964	0.961	0.957	0.959

Table S3 continued from previous page

	power	0.935	0.917	0.911	0.913	0.928
	logistic2	0.969	0.961	0.954	0.950	0.953
	betaMod	0.488	0.703	0.720	0.767	0.649
$N = 60$	linear	0.972	0.968	0.963	0.954	0.960
	emax1	0.966	0.968	0.962	0.955	0.960
	emax2	0.923	0.937	0.927	0.943	0.955
	exponential1	0.932	0.931	0.930	0.906	0.951
	quadratic1	0.850	0.900	0.897	0.883	0.911
	logistic1	0.987	0.982	0.975	0.967	0.984
	exponential2	0.957	0.949	0.949	0.934	0.953
	quadratic2	0.950	0.952	0.934	0.943	0.955
	sigEmax	0.986	0.984	0.979	0.980	0.977
	power	0.964	0.955	0.948	0.945	0.943
	logistic2	0.984	0.979	0.980	0.967	0.973
	betaMod	0.604	0.781	0.802	0.825	0.708

Table S3: Power values of LiMAP-curvature, smoothing spline and MCP-Mod to test the PoC across different true underlying dose-response models and sample sizes by controlling the type I error rate at 10%.

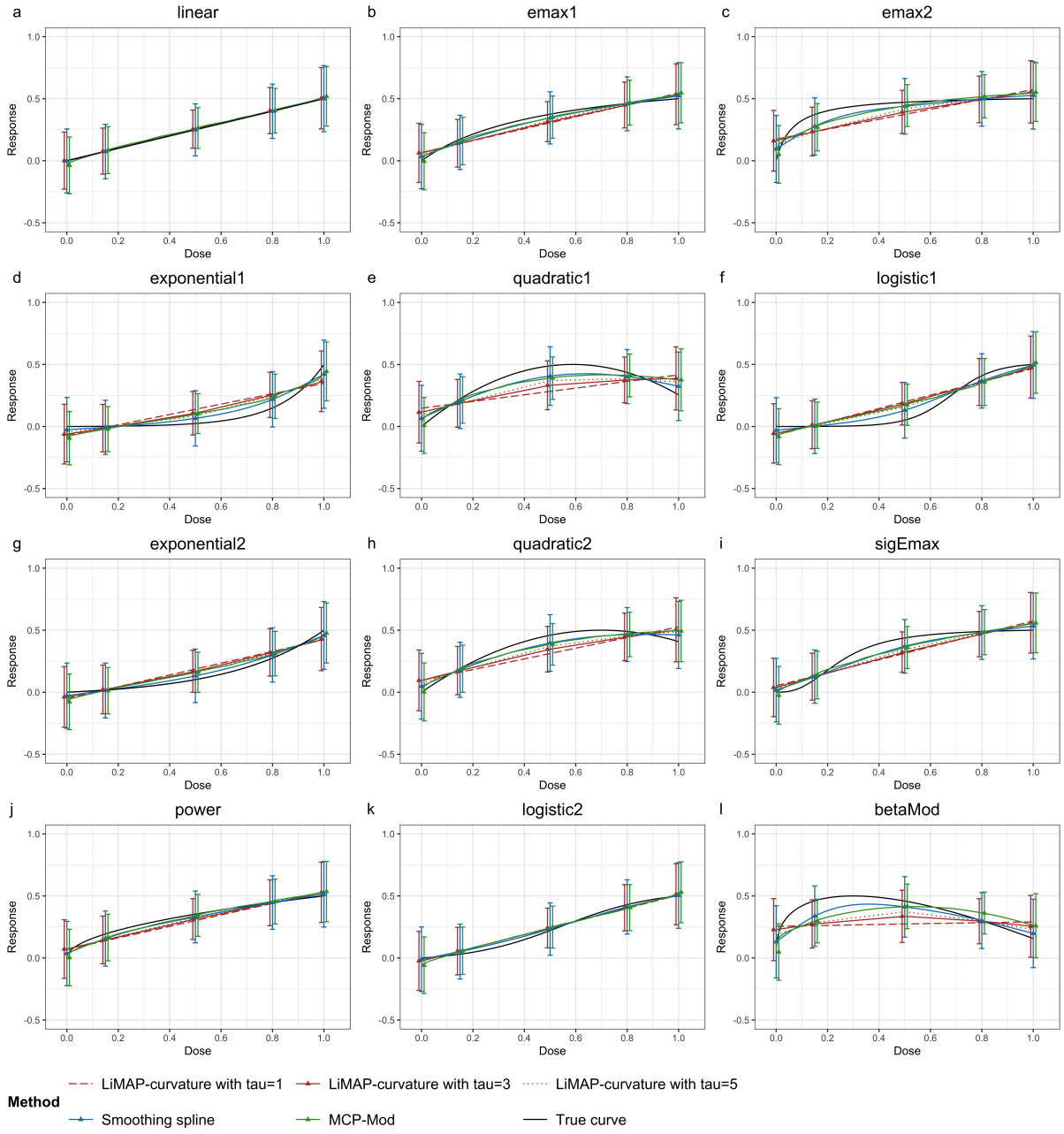

Figure S6: Dose-response curves estimated with LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 10 patients per arm. The dose-response curves in (a)-(f) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set, and the dose-response curves in (g)-(l) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set.

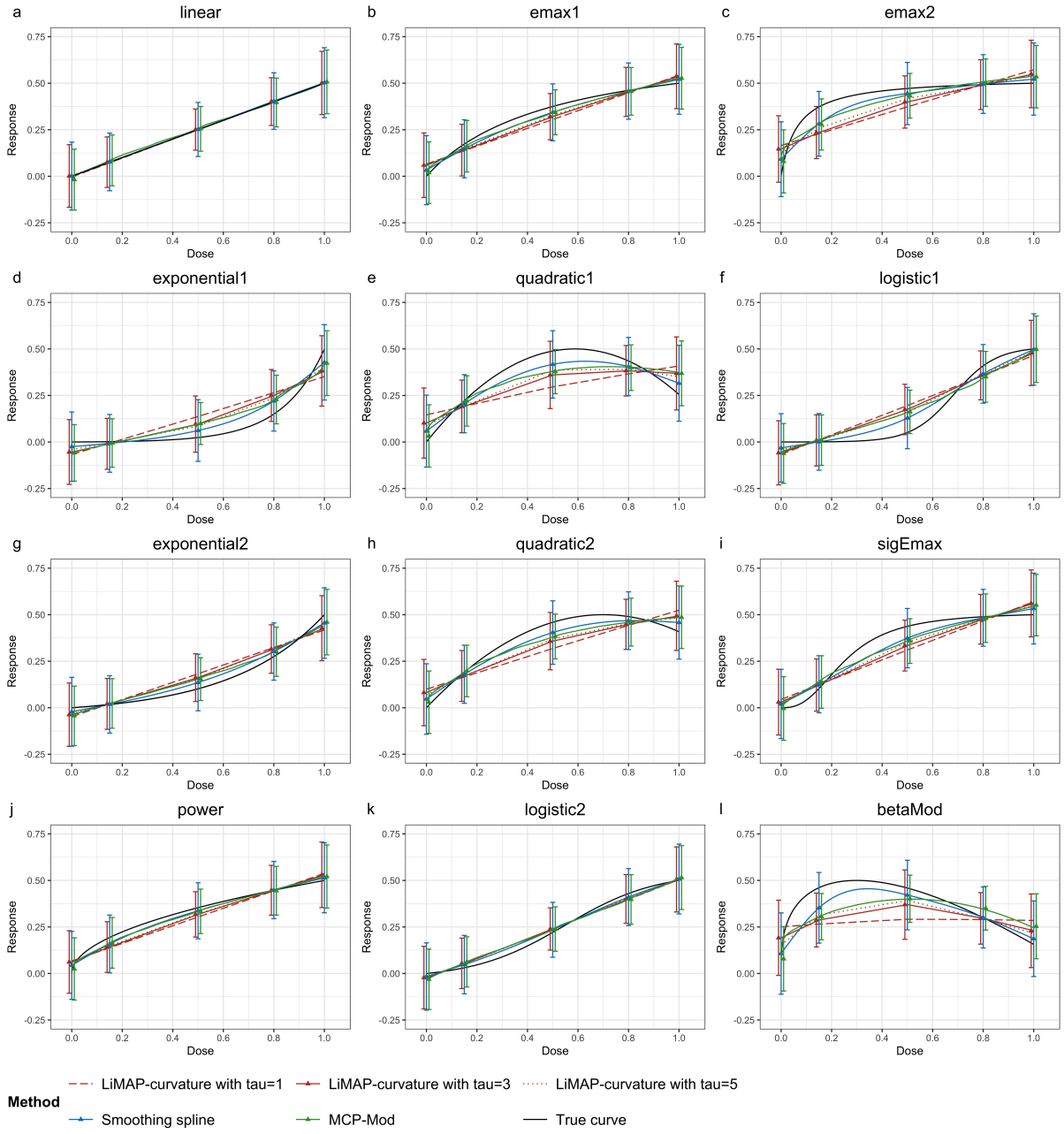

Figure S7: Dose-response curves estimated with LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 20 patients per arm. The dose-response curves in (a)-(f) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set, and the dose-response curves in (g)-(l) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set.

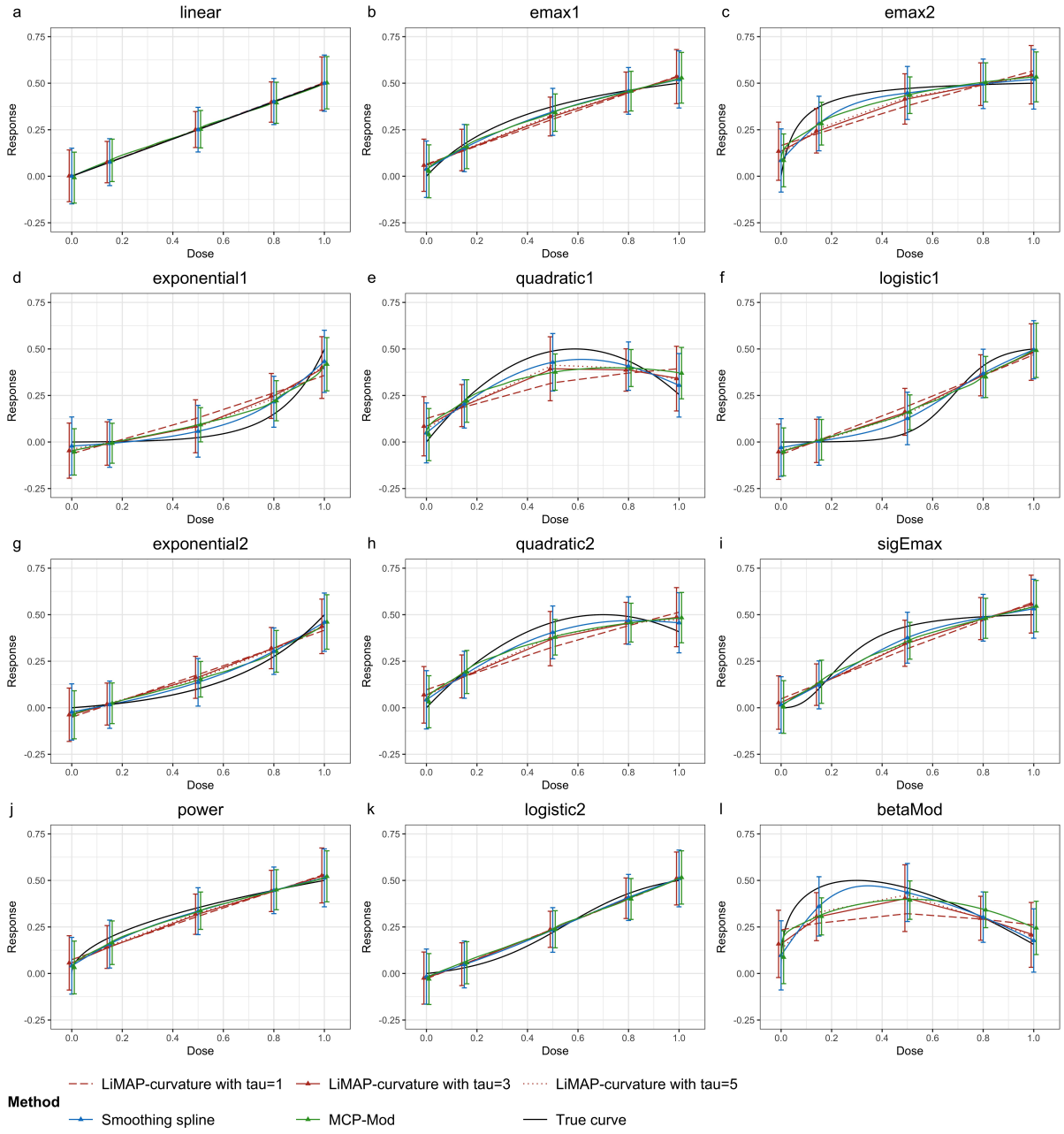

Figure S8: Dose-response curves estimated with LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 30 patients per arm. The dose-response curves in (a)-(f) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set, and the dose-response curves in (g)-(l) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set.

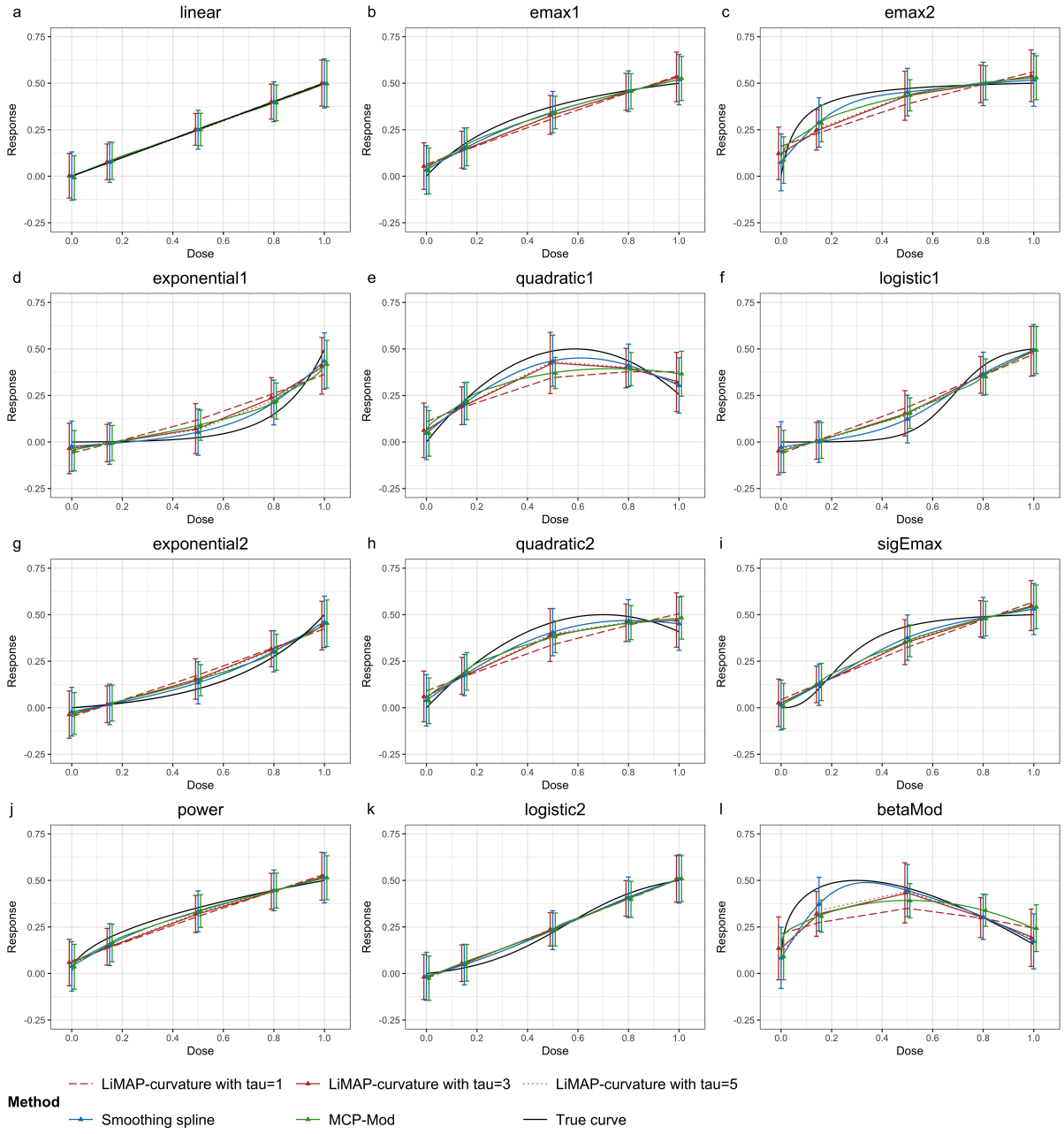

Figure S9: Dose-response curves estimated with LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 40 patients per arm. The dose-response curves in (a)-(f) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set, and the dose-response curves in (g)-(l) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set.

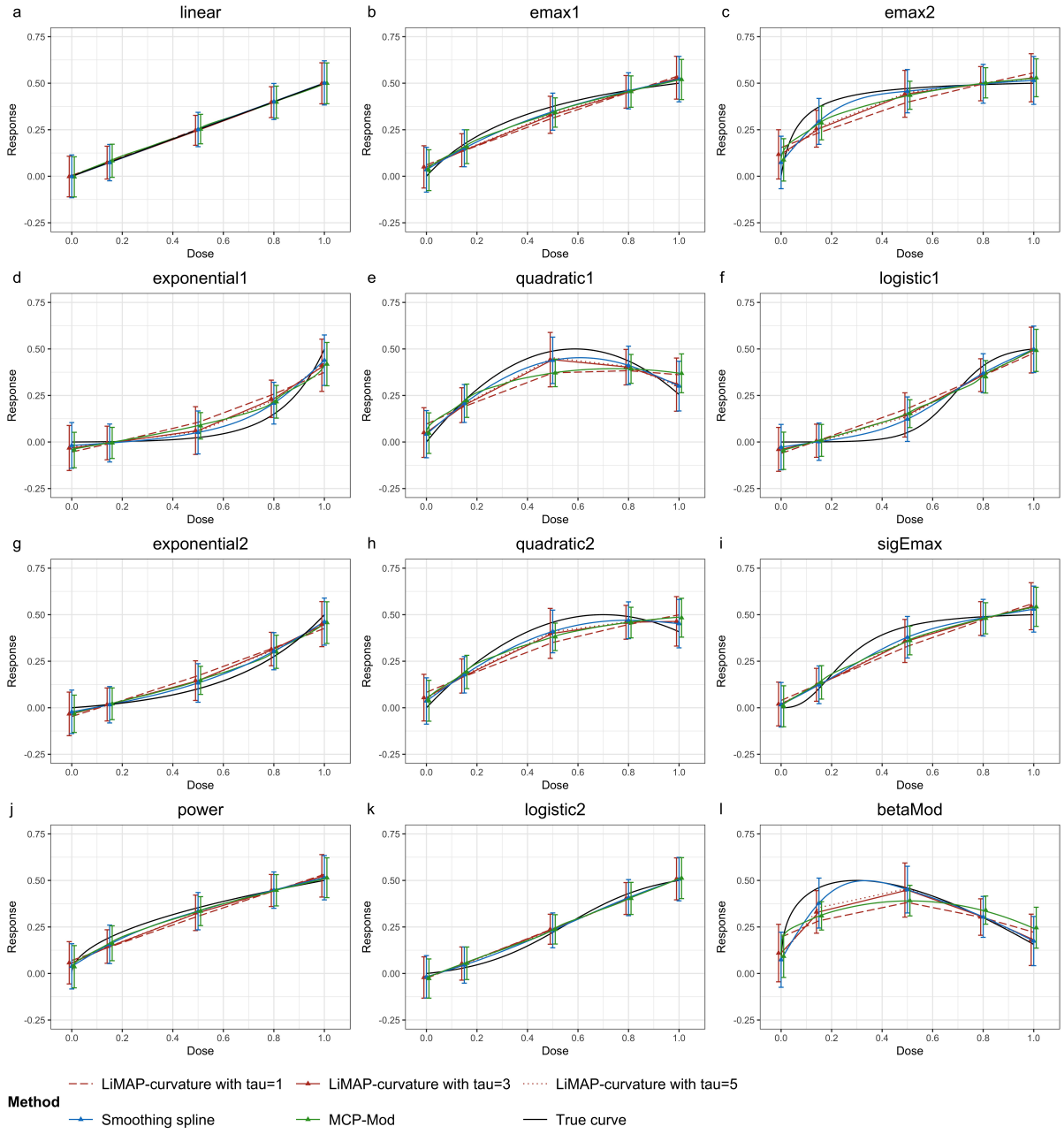

Figure S10: Dose-response curves estimated with LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 50 patients per arm. The dose-response curves in (a)-(f) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set, and the dose-response curves in (g)-(l) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set.

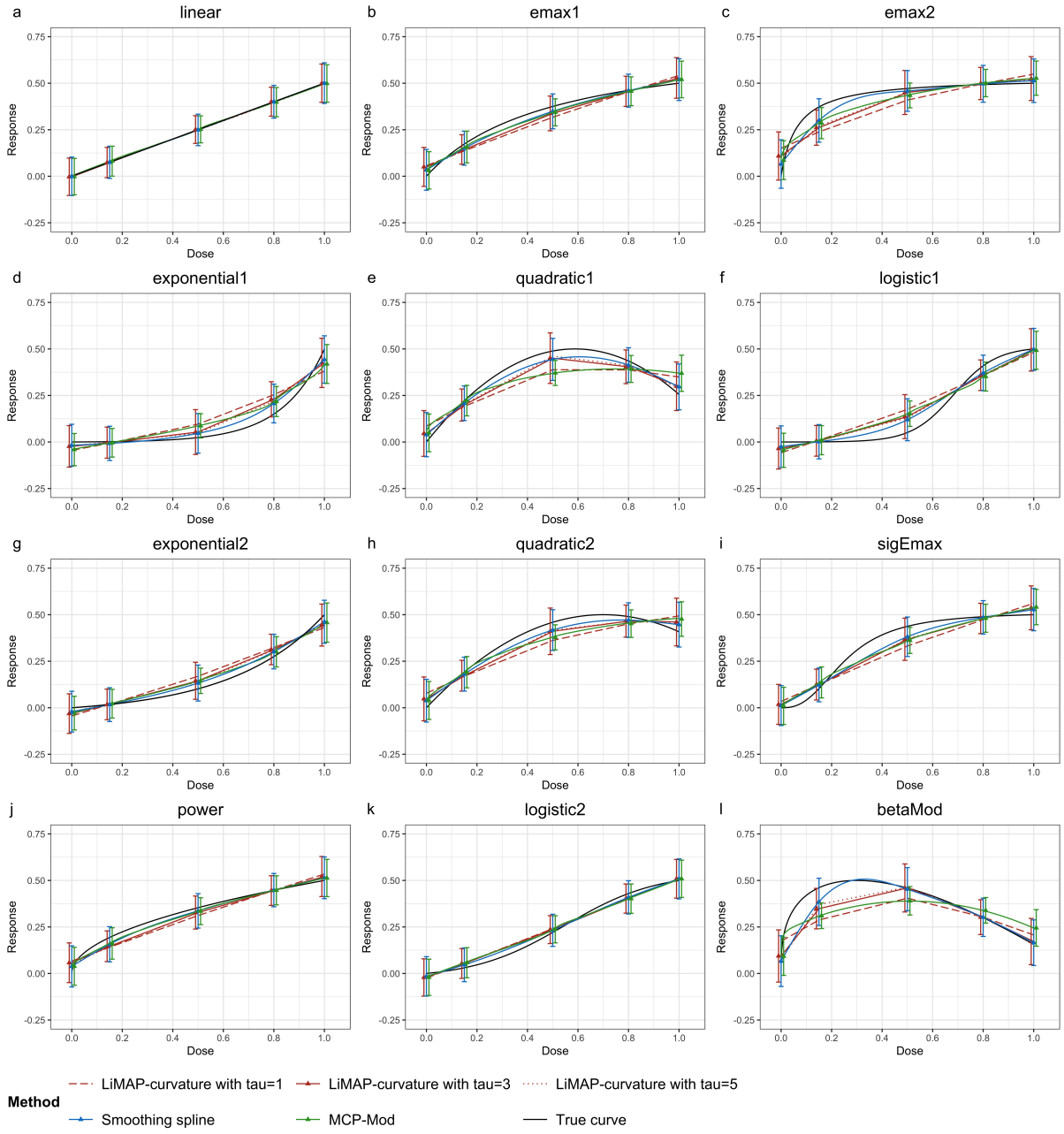

Figure S11: Dose-response curves estimated with LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models with the sample size of 60 patients per arm. The dose-response curves in (a)-(f) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set, and the dose-response curves in (g)-(l) are produced through MCP-Mod with the true underlying dose-response model not included in the candidate model set.

Sample size	Model	True MED	LiMAP-curvature						Smoothing spline		MCP-Mod	
			$\tau = 1$		$\tau = 3$		$\tau = 5$					
			Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$N = 10$	linear	0.600	-0.098	0.046	-0.098	0.048	-0.100	0.053	-0.139	0.071	-0.120	0.089
	emax1	0.334	0.175	0.067	0.168	0.070	0.158	0.068	0.079	0.052	0.074	0.079
	emax2	0.083	0.450	0.239	0.430	0.229	0.407	0.216	0.303	0.162	0.238	0.136
	exponential1	0.916	-0.385	0.186	-0.365	0.176	-0.367	0.180	-0.325	0.170	-0.268	0.143
	quadratic1	0.216	0.357	0.165	0.314	0.149	0.260	0.124	0.135	0.072	0.078	0.065
	logistic1	0.713	-0.217	0.084	-0.211	0.084	-0.208	0.085	-0.180	0.080	-0.168	0.091
	exponential2	0.828	-0.316	0.137	-0.303	0.132	-0.309	0.136	-0.343	0.185	-0.261	0.143
	quadratic2	0.257	0.273	0.114	0.252	0.107	0.236	0.102	0.114	0.080	0.084	0.072
	sigEmax	0.291	0.209	0.081	0.190	0.077	0.185	0.074	0.139	0.082	0.100	0.075
	power	0.360	0.153	0.060	0.149	0.063	0.137	0.062	0.096	0.076	0.045	0.081
	logistic2	0.601	-0.109	0.048	-0.104	0.049	-0.112	0.051	-0.143	0.071	-0.117	0.081
	betaMod	0.075	0.545	0.341	0.447	0.268	0.355	0.217	0.135	0.059	0.141	0.080
$N = 20$	linear	0.600	-0.049	0.035	-0.052	0.036	-0.059	0.039	-0.077	0.055	-0.082	0.067
	emax1	0.334	0.225	0.083	0.211	0.081	0.201	0.081	0.123	0.072	0.099	0.073
	emax2	0.083	0.512	0.295	0.470	0.268	0.438	0.252	0.300	0.156	0.258	0.134
	exponential1	0.916	-0.327	0.141	-0.303	0.129	-0.296	0.129	-0.241	0.117	-0.196	0.087
	quadratic1	0.216	0.417	0.213	0.317	0.165	0.295	0.156	0.124	0.067	0.083	0.051
	logistic1	0.713	-0.178	0.063	-0.165	0.061	-0.165	0.063	-0.135	0.067	-0.100	0.055
	exponential2	0.828	-0.260	0.101	-0.256	0.101	-0.248	0.099	-0.231	0.106	-0.186	0.088
	quadratic2	0.257	0.322	0.139	0.292	0.132	0.271	0.122	0.165	0.082	0.102	0.061

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	sigEmax	0.291	0.250	0.095	0.227	0.086	0.219	0.088	0.142	0.071	0.109	0.063
	power	0.360	0.210	0.077	0.198	0.078	0.184	0.077	0.095	0.069	0.095	0.077
	logistic2	0.601	-0.069	0.036	-0.066	0.038	-0.069	0.038	-0.101	0.055	-0.075	0.059
	betaMod	0.075	0.586	0.392	0.332	0.191	0.267	0.154	0.117	0.044	0.136	0.055
$N = 30$	linear	0.600	-0.026	0.029	-0.031	0.032	-0.036	0.034	-0.052	0.049	-0.053	0.054
	emax1	0.334	0.254	0.095	0.227	0.088	0.220	0.089	0.145	0.078	0.118	0.070
	emax2	0.083	0.532	0.317	0.475	0.279	0.442	0.259	0.288	0.154	0.257	0.123
	exponential1	0.916	-0.287	0.114	-0.259	0.103	-0.258	0.105	-0.209	0.094	-0.149	0.057
	quadratic1	0.216	0.391	0.200	0.280	0.152	0.266	0.145	0.108	0.057	0.096	0.047
	logistic1	0.713	-0.156	0.052	-0.141	0.052	-0.136	0.051	-0.104	0.051	-0.086	0.043
	exponential2	0.828	-0.234	0.084	-0.223	0.082	-0.225	0.084	-0.195	0.088	-0.154	0.066
	quadratic2	0.257	0.346	0.154	0.295	0.136	0.281	0.130	0.128	0.065	0.121	0.063
	sigEmax	0.291	0.265	0.100	0.235	0.090	0.230	0.091	0.154	0.065	0.117	0.057
	power	0.360	0.235	0.086	0.216	0.082	0.201	0.079	0.126	0.071	0.096	0.066
	logistic2	0.601	-0.045	0.030	-0.047	0.032	-0.046	0.033	-0.056	0.044	-0.045	0.046
	betaMod	0.075	0.500	0.318	0.210	0.100	0.189	0.107	0.093	0.032	0.131	0.044
$N = 50$	linear	0.600	-0.006	0.024	-0.005	0.027	-0.004	0.028	-0.031	0.039	-0.031	0.042
	emax1	0.334	0.265	0.097	0.233	0.089	0.233	0.092	0.160	0.072	0.131	0.063
	emax2	0.083	0.536	0.326	0.448	0.265	0.429	0.255	0.255	0.127	0.268	0.123
	exponential1	0.916	-0.243	0.085	-0.206	0.073	-0.206	0.074	-0.152	0.061	-0.114	0.035
	quadratic1	0.216	0.317	0.159	0.192	0.099	0.166	0.088	0.095	0.041	0.111	0.044
	logistic1	0.713	-0.133	0.042	-0.110	0.039	-0.108	0.040	-0.064	0.034	-0.056	0.028

Table S4 continued from previous page

	exponential2	0.828	-0.199	0.066	-0.185	0.062	-0.182	0.063	-0.155	0.061	-0.120	0.046
	quadratic2	0.257	0.338	0.154	0.266	0.124	0.254	0.122	0.140	0.064	0.124	0.052
	sigEmax	0.291	0.264	0.097	0.232	0.089	0.230	0.088	0.152	0.060	0.131	0.052
	power	0.360	0.253	0.091	0.231	0.089	0.222	0.086	0.130	0.073	0.120	0.064
	logistic2	0.601	-0.030	0.024	-0.025	0.026	-0.029	0.026	-0.029	0.036	-0.032	0.034
	betaMod	0.075	0.312	0.139	0.146	0.046	0.099	0.028	0.076	0.017	0.151	0.042
$N = 60$	linear	0.600	-0.046	0.035	-0.048	0.036	-0.054	0.038	-0.020	0.036	-0.018	0.037
	emax1	0.334	0.227	0.084	0.205	0.078	0.198	0.080	0.149	0.070	0.136	0.060
	emax2	0.083	0.506	0.290	0.464	0.262	0.424	0.240	0.233	0.116	0.273	0.119
	exponential1	0.916	-0.328	0.141	-0.307	0.132	-0.299	0.131	-0.136	0.050	-0.104	0.029
	quadratic1	0.216	0.401	0.200	0.315	0.159	0.283	0.152	0.089	0.037	0.112	0.042
	logistic1	0.713	-0.180	0.064	-0.165	0.062	-0.161	0.062	-0.059	0.030	-0.049	0.025
	exponential2	0.828	-0.259	0.100	-0.250	0.098	-0.253	0.103	-0.139	0.053	-0.110	0.040
	quadratic2	0.257	0.328	0.142	0.288	0.127	0.269	0.122	0.130	0.060	0.135	0.054
	sigEmax	0.291	0.249	0.094	0.226	0.087	0.217	0.086	0.147	0.056	0.133	0.048
	power	0.360	0.212	0.078	0.193	0.073	0.179	0.073	0.132	0.071	0.137	0.064
	logistic2	0.601	-0.061	0.036	-0.064	0.037	-0.069	0.039	-0.025	0.029	-0.023	0.031
	betaMod	0.075	0.603	0.406	0.342	0.207	0.221	0.124	0.072	0.014	0.154	0.042

Table S4: Bias and Mean Square Error (MSE) of minimum effective doses (MED's) estimated through LiMAP-curvature, smoothing spline and MCP-Mod across different true underlying dose-response models and sample sizes with the clinical relevance threshold 0.3.

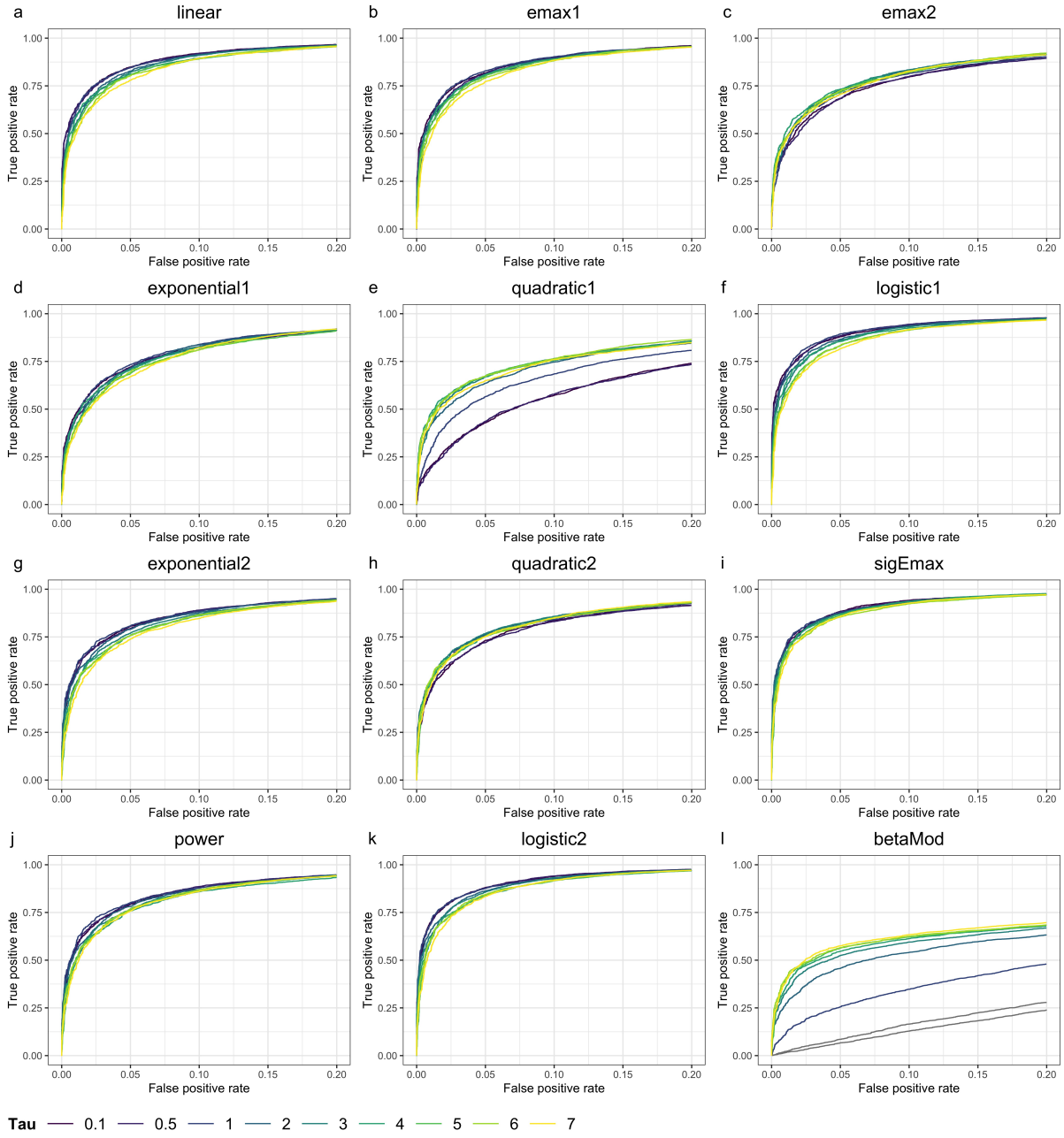

Figure S12: ROC curves of LiMAP-curvature with varying values of τ across different true underlying dose-response models with the sample size of 40 patients per arm.

Model	True MED	LiMAP-curvature																	
		$\tau = 0.1$		$\tau = 0.5$		$\tau = 1$		$\tau = 2$		$\tau = 3$		$\tau = 4$		$\tau = 5$		$\tau = 6$		$\tau = 7$	
		Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
linear	0.600	-0.016	0.026	-0.012	0.026	-0.013	0.026	-0.013	0.028	-0.015	0.029	-0.017	0.029	-0.016	0.031	-0.019	0.031	-0.018	0.030
emax1	0.334	0.265	0.096	0.264	0.097	0.258	0.095	0.244	0.091	0.236	0.090	0.233	0.091	0.233	0.092	0.230	0.092	0.228	0.091
emax2	0.083	0.571	0.353	0.564	0.346	0.537	0.323	0.493	0.295	0.463	0.273	0.450	0.267	0.439	0.261	0.435	0.258	0.435	0.258
exponential1	0.916	-0.281	0.107	-0.279	0.105	-0.261	0.097	-0.243	0.091	-0.238	0.089	-0.230	0.087	-0.229	0.088	-0.227	0.087	-0.230	0.090
quadratic1	0.216	0.498	0.273	0.484	0.264	0.364	0.190	0.263	0.138	0.234	0.123	0.221	0.119	0.218	0.119	0.208	0.116	0.197	0.111
logistic1	0.713	-0.148	0.048	-0.149	0.048	-0.141	0.047	-0.137	0.047	-0.127	0.046	-0.120	0.044	-0.120	0.044	-0.119	0.044	-0.119	0.044
exponential2	0.828	-0.220	0.076	-0.221	0.076	-0.214	0.074	-0.207	0.073	-0.203	0.072	-0.200	0.073	-0.199	0.071	-0.201	0.073	-0.201	0.073
quadratic2	0.257	0.379	0.171	0.378	0.171	0.346	0.155	0.300	0.138	0.283	0.131	0.270	0.127	0.269	0.128	0.268	0.129	0.264	0.127
sigEmax	0.291	0.278	0.103	0.278	0.103	0.267	0.100	0.248	0.095	0.237	0.092	0.233	0.091	0.230	0.089	0.232	0.091	0.228	0.090
power	0.360	0.251	0.090	0.249	0.089	0.244	0.088	0.232	0.086	0.228	0.087	0.223	0.086	0.223	0.087	0.218	0.086	0.212	0.084
logistic2	0.601	-0.037	0.027	-0.037	0.027	-0.037	0.027	-0.032	0.028	-0.035	0.029	-0.032	0.029	-0.033	0.029	-0.036	0.029	-0.032	0.029
betaMod	0.075	0.721	0.540	0.661	0.478	0.375	0.201	0.233	0.103	0.183	0.079	0.156	0.066	0.142	0.063	0.139	0.062	0.132	0.061

Table S5: Bias and Mean Square Error (MSE) of minimum effective doses (MED's) estimated through LiMAP-curvature with varying values of τ across different true underlying dose-response models with the sample size of 40 patients per arm with the clinical relevance threshold 0.3.